Cuaderno de notas de trabajo

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Cuaderno Sn 17

Símbolos de Birkhoff de Primera Especie . Campo Central

$$\boxed{B_{ij,k}}$$

Coordenadas Cartesianas.

Cuaderno 33.

Página 1. Cuaderno 33

$$\frac{d^2x^i}{ds^2} + \Delta^{im} B_{jk,m} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0 ;$$

$$a^i + \Delta^{im} B_{jk,m} v^j v^k = 0 .$$

$$\boxed{a_i + B_{jk,i} v^j v^k = 0}$$

Calcular $\boxed{B_{jk,i} v^j = D_{ki}}$

$$\boxed{D_{ki} = B_{jk,i}\, v^j}$$

cuaderno 33
páginas 3, 4

$$D_{11} = \frac{M}{2r^3}\left(xx' + yy' + zz'\right)$$

$$D_{21} = \frac{M}{2r^3}\left(\dot{x}t'\right)$$

$$D_{31} = \frac{M}{2r^3}\left(yt'\right)$$

$$D_{41} = \frac{M}{2r^3}\left(zt'\right)$$

$$D_{12} = -\frac{M}{r^3}\left(xt'\right) \qquad D_{14} = -\frac{M}{r^3}\left(zt'\right)$$

$$D_{22} = \frac{M}{2r^3}\left(yy' + zz'\right) \qquad D_{24} = +\frac{M}{2r^3}\left(-2zx' + xz'\right)$$

$$D_{32} = \frac{M}{2r^3}\left(yx' - 2xy'\right) \qquad D_{34} = \frac{M}{2r^3}\left(-2zy' + yz'\right)$$

$$D_{42} = \frac{M}{2r^3}\left(zx' - 2xz'\right) \qquad D_{44} = \frac{M}{2r^3}\left(xx' + yy'\right)$$

$$D_{13} = -\frac{M}{r^3}\left(yt'\right)$$

$$D_{23} = +\frac{M}{2r^3}\left(-2yx' + xy'\right)$$

$$D_{33} = \frac{M}{2r^3}\left(xx' + zz'\right)$$

$$D_{43} = \frac{M}{2r^3}\left(zy' - 2yz'\right)$$

Campo Central

$$D_{ki}$$

$$\begin{array}{c|c|c|c}
\frac{M}{2r^3} r r' & -\frac{Mt'}{r^3} x & -\frac{Mt'}{r^3} y & -\frac{Mt'}{r^3} z \\
\hline
\frac{Mt'}{2r^3} x & \frac{M}{2r^3}(rr' - xx') & \frac{M}{2r^3}(-2yx' + xy') & \frac{M}{2r^3}(-2zx' + xz') \\
\hline
\frac{Mt'}{2r^3} y & \frac{M}{2r^3}(yx' - 2xy') & \frac{M}{2r^3}(rr' - yy') & \frac{M}{2r^3}(-2zy' + yz') \\
\hline
\frac{Mt'}{2r^3} z & \frac{M}{2r^3}(zx' - 2xz') & \frac{M}{2r^3}(zy' - 2yz') & \frac{M}{2r^3}(rr' - zz')
\end{array}$$

$$\boxed{B_{jk,i} = \frac{\partial h_{jk}}{\partial x^i} - \frac{1}{2}\frac{\partial h_{ij}}{\partial x^k} - \frac{1}{2}\frac{\partial h_{ik}}{\partial x^j}}$$

Cuaderno 33 página 1.

$$\boxed{D_{ki} = B_{jk,i}\, v^j}$$

Campo Central.

Orbits in Birkhoff Central Field.
página 169.

$$x'' = -\frac{Mx}{r^3} - \frac{2Mx}{r^3}(x'^2+y'^2+z'^2) + \frac{Mx'r'}{r^2}$$

$$t'' = -\frac{Mt'r'}{r^2}$$

$$t'' = -\frac{Mt'r'}{r^2}$$

$$x'' = -\frac{Mx}{r^3} - \frac{2Mx}{r^3}(x'^2+y'^2+z'^2) + \frac{Mx'r'}{r^2}$$

$$y'' = -\frac{My}{r^3} - \frac{2My}{r^3}(x'^2+y'^2+z'^2) + \frac{My'r'}{r^2}$$

$$z'' = -\frac{Mz}{r^3} - \frac{2Mz}{r^3}(x'^2+y'^2+z'^2) + \frac{Mz'r'}{r^2}$$

Campo Central

$\boxed{\hat{a}}$ cuadrivector aceleración.

Componentes covariantes.

$$a_1 = -\frac{M t' r'}{r^2}$$

$t'-1$

$$a_2 = +\frac{Mx}{r^3} + \frac{2Mx}{r^3}\left(x'^2+y'^2+z'^2\right) - \frac{M x' r'}{r^2}$$

$$a_3 = +\frac{My}{r^3} + \frac{2My}{r^3}\left(x'^2+y'^2+z'^2\right) - \frac{M y' r'}{r^2}$$

$$a_4 = +\frac{Mz}{r^3} + \frac{2Mz}{r^3}\left(x'^2+y'^2+z'^2\right) - \frac{M z' r'}{r^2}$$

Cuadrivector velocidad $\hat{v}$

Componentes covariantes.

$$v_1 = t'$$

$$v_2 = -x'$$

$$v_3 = -y'$$

$$v_4 = -z'$$

$$\boxed{t'^2 - x'^2 - y'^2 - z'^2 = 1}$$

$$\boxed{x'^2 + y'^2 + z'^2 = t'^2 - 1}$$

Calcular $a_k v_i$ y $a_i v_k$

$$\boxed{\begin{array}{c} \alpha_{ki} = a_k v_i \\ \hline \beta_{ki} = a_i v_k \end{array}}$$

$\Leftarrow$ ojo

D_{ki} Ver página 3 de este cuaderno.

$S(D_{ki})$ = parte simétrica de D_{ki}

$A(D_{ki})$ = parte antisimétrica de D_{ki}

$S(D_{ki})$ Parte simétrica de D_{ki}

$\frac{M}{2r^3} rr'$	$-\frac{Mt'}{4r^3} x$	$-\frac{Mt'}{4r^3} y$	$-\frac{Mt'}{4r^3} z$
$-\frac{Mt'}{4r^3} x$	$\frac{M}{2r^3}(rr' - xx')$	$-\frac{M}{2r^3}(xy' + yx')$	$-\frac{M}{2r^3}(xz' + zx')$
$-\frac{Mt'}{4r^3} y$	$-\frac{M}{2r^3}(xy' + yx')$	$\frac{M}{2r^3}(rr' - yy')$	$-\frac{M}{2r^3}(yz' + zy')$
$-\frac{Mt'}{4r^3} z$	$-\frac{M}{2r^3}(xz' + zx')$	$-\frac{M}{2r^3}(yz' + zy')$	$\frac{M}{2r^3}(rr' - zz')$

Parte simétrica de D_{ki}

$\mathcal{A}(D_{ki})$ Parte antisimétrica de D_{ki}

$$\begin{pmatrix}
0 & -\frac{3Mt'}{4r^3}x & -\frac{3Mt'}{4r^3}y & -\frac{3Mt'}{4r^3}z \\
+\frac{3Mt'}{4r^3}x & 0 & \frac{3M}{4r^3}(xy'-yx') & \frac{3M}{4r^3}(xz'-zx') \\
+\frac{3Mt'}{4r^3}y & -\frac{3M}{4r^3}(xy'-yx') & 0 & \frac{3M}{4r^3}(yz'-zy') \\
+\frac{3Mt'}{4r^3}z & -\frac{3M}{4r^3}(xz'-zx') & -\frac{3M}{4r^3}(yz'-zy') & 0
\end{pmatrix}$$

$$\boxed{D_{ki} = \mathcal{S}(D_{ki}) + \mathcal{A}(D_{ki})}$$

$$A(D_{ki}) = \frac{1}{2}(D_{ki} - D_{ik})$$

Consideraciones sobre las componentes covariantes del cuadri vector aceleración:

Ver página 5.

$$\boxed{t'^2 - x'^2 - y'^2 - z'^2 = 1}$$

$$\boxed{x'^2 + y'^2 + z'^2 = t'^2 - 1}$$

$$a_1 = -\frac{M t' r'}{r^2}$$

$$a_2 = -\frac{Mx}{r^3} + \frac{2Mx}{r^3} t'^2 - \frac{M x' r'}{r^2}$$

$$a_3 = -\frac{My}{r^3} + \frac{2My}{r^3} t'^2 - \frac{M y' r'}{r^2}$$

$$a_4 = -\frac{Mz}{r^3} + \frac{2Mz}{r^3} t'^2 - \frac{M z' r'}{r^2}$$

$$v_1 = t'$$

$$v_2 = -x'$$

$$v_3 = -y'$$

$$v_4 = -z'$$

$\mathcal{T}$ — Traza de

$$\mathcal{T}(a_i v_j)$$

$$+\frac{M}{r^3}\left[-rr't'^2 - xx' - yy' - zz' + 2xx't'^2 + 2yy't'^2 + 2zz't'^2 - rr'x'^2 - rr'y'^2 - rr'z'^2\right].$$

$$\cancel{\mathcal{T}(a_i v_j) = \frac{M}{r^3}\left[+rr' + rr' - 2rr't'^2\right]}$$

$$\mathcal{T}(a_i v_j) = \left[-rr't'^2 - rr' + 2rr't'^2 - rr'\{1 - t'^2\}\right]$$

$$\mathcal{T}(a_i v_j) = \left[-2rr' + 2rr't'^2\right]\frac{M}{r^3}$$

$$\boxed{\mathcal{T}(a_i v_j) = \frac{2M}{r^2}r'(x'^2 + y'^2 + z'^2)}$$

$\boxed{\alpha_{ij}}$

$\boxed{a_i \; v_j}$

Ver página 6

$\boxed{\alpha_{ij}}$ $\qquad \tau(a_i v_j) = \left[-2rr' + 2rr't'^2\right]\frac{M}{r^3}$

$$\boxed{\tau(a_i v_j) = \frac{2M r'}{r^2}(x'^2 + y'^2 + z'^2)}$$

$\boxed{a_i\ V_j}$

Ver página 6

$-\frac{Mr'}{r^2}t'^2$	$+\frac{Mr'x'}{r^2}t'$	$+\frac{Mr'y'}{r^2}t'$	$+\frac{Mr'z'}{r^2}t'$
$-\frac{Mx}{r^3}t'$	$+\frac{Mxx'}{r^3}$	$+\frac{Mxy'}{r^3}$	$+\frac{Mxz'}{r^3}$
$+\frac{2Mx}{r^3}t'^3$	$-\frac{2Mxx'}{r^3}t'^2$	$-\frac{2Mxy'}{r^3}t'^2$	$-\frac{2Mxz'}{r^3}t'^2$
$-\frac{Mx'r'}{r^2}t'$	$+\frac{Mx'^2r'}{r^2}$	$+\frac{Mx'y'r'}{r^2}$	$+\frac{Mx'z'r'}{r^2}$
$-\frac{My}{r^3}t'$	$+\frac{Myx'}{r^3}$	$+\frac{Myy'}{r^3}$	$+\frac{Myz'}{r^3}$
$+\frac{2My}{r^3}t'^3$	$-\frac{2Myx'}{r^3}t'^2$	$-\frac{2Myy'}{r^3}t'^2$	$-\frac{2Myz'}{r^3}t'^2$
$-\frac{My'r'}{r^2}t'$	$+\frac{My'x'r'}{r^2}$	$+\frac{My'^2r'}{r^2}$	$+\frac{My'z'r'}{r^2}$
$-\frac{Mz}{r^3}t'$	$+\frac{Mzx'}{r^3}$	$+\frac{Mzy'}{r^3}$	$+\frac{Mzz'}{r^3}$
$+\frac{2Mz}{r^3}t'^3$	$-\frac{2Mzx'}{r^3}t'^2$	$-\frac{2Mzy'}{r^3}t'^2$	$-\frac{2Mzz'}{r^3}t'^2$
$-\frac{Mz'r'}{r^2}t'$	$+\frac{Mz'x'r'}{r^2}$	$+\frac{My'z'r'}{r^2}$	$+\frac{Mz'^2r'}{r^2}$

$$\boxed{B_{ij}}$$

$$\boxed{B_{ij} = a_j v_i}$$ Ver página 6.

$-\frac{Mr'}{r^2}t'^2$	$-\frac{Mx}{r^3}t'$ $+\frac{2Mx}{r^3}t'^3$ $-\frac{Mx'r'}{r^2}t'$	$-\frac{My}{r^3}t'$ $+\frac{2My}{r^3}t'^3$ $-\frac{My'r'}{r^2}t'$	$-\frac{Mz}{r^3}t'$ $+\frac{2Mz}{r^3}t'^3$ $-\frac{Mz'r'}{r^2}t'$
$+\frac{Mr'x'}{r^2}t'$	$+\frac{Mxx'}{r^3}$ $-\frac{2Mxx'}{r^3}t'^2$ $+\frac{Mx'^2r'}{r^2}$	$+\frac{Myx'}{r^3}$ $-\frac{2Myx'}{r^3}t'^2$ $+\frac{My'x'r'}{r^2}$	$+\frac{Mzx'}{r^3}$ $-\frac{2Mzx'}{r^3}t'^2$ $+\frac{Mz'x'r'}{r^2}$
$+\frac{Mr'y'}{r^2}t'$	$+\frac{Mxy'}{r^3}$ $-\frac{2Mxy'}{r^3}t'^2$ $+\frac{Mx'y'r'}{r^2}$	$+\frac{Myy'}{r^3}$ $-\frac{2Myy'}{r^3}t'^2$ $+\frac{My'^2r'}{r^2}$	$+\frac{Mzy'}{r^3}$ $-\frac{2Mzy'}{r^3}t'^2$ $+\frac{My'z'r'}{r^2}$
$+\frac{Mr'z'}{r^2}t'$	$+\frac{Mxz'}{r^3}$ $-\frac{2Mxz'}{r^3}t'^2$ $+\frac{Mx'z'r'}{r^2}$	$+\frac{Myz'}{r^3}$ $-\frac{2Myz'}{r^3}t'^2$ $+\frac{My'z'r'}{r^2}$	$+\frac{Mzz'}{r^3}$ $-\frac{2Mzz'}{r^3}t'^2$ $+\frac{Mz'^2r'}{r^2}$

Líneas de universo de las partículas exploradoras:

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left(\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{jm}}{\partial x^k}\right) v^j v^k = 0$$

$$a_i + \left(\frac{\partial h_{jk}}{\partial x^i} - \frac{\partial h_{ij}}{\partial x^k}\right) v^j v^k = 0$$

$$\boxed{a_i + \mathcal{B}_{jk,i}\, v^j v^k = 0}$$

Ver cuaderno 33 pág. 05, 73, 74,

$$\boxed{\mathcal{B}_{jk,i} v^j = L_{ki}}$$

$$\boxed{\mathcal{B}_{jk,i} v^k = M_{ji}}$$

$$\boxed{L_{ki} = B_{jk,i} v^j}$$

$$\boxed{M_{ji} = B_{jk,i} v^k}$$

$$\boxed{L_{ki} = B_{jk,i}\, v^j}$$ ¡Antisimétrico!

Comprobado en la página 29.

0	$-\frac{Mx}{r^3}t'$	$-\frac{My}{r^3}t'$	$-\frac{Mz}{r^3}t'$
$\frac{Mx}{r^3}t'$	0	$\frac{M}{r^3}(-yx'+xy')$	$\frac{M}{r^3}(-zx'+xz')$
$\frac{My}{r^3}t'$	$\frac{M}{r^3}(yx'-xy')$	0	$\frac{M}{r^3}(-zy'+yz')$
$\frac{Mz}{r^3}t'$	$\frac{M}{r^3}(zx'-xz')$	$\frac{M}{r^3}(zy'-yz')$	0

$$\boxed{M_{ki} = \mathcal{B}_{kj,i}\, v^j}$$

Comprobado en la página 32.

$\frac{M}{r^2} r'$	$-\frac{Mx}{r^3} t'$	$-\frac{My}{r^3} t'$	$-\frac{Mz}{r^3} t'$
0	$\frac{M}{r^3}(yy'+zz')$	$-\frac{Myx'}{r^3}$	$-\frac{Mzx'}{r^3}$
0	$-\frac{Mxy'}{r^3}$	$\frac{M}{r^3}(xx'+zz')$	$-\frac{Mzy'}{r^3}$
0	$-\frac{Mxz'}{r^3}$	$-\frac{Myz'}{r^3}$	$\frac{M}{r^3}(xx'+yy')$

$\mathcal{A}$ — parte antisimétrica.

$$\mathcal{A}(a_i v_j) = \frac{1}{2}(a_i v_j - a_j v_i)$$

$$\mathcal{A}(a_i v_j) = \frac{1}{2}\alpha_{ij} - \frac{1}{2}\beta_{ij}$$

$$\boxed{\mathcal{A}(a_i v_j) = \frac{1}{2}(\alpha_{ij} - \beta_{ij})}$$

$$
\begin{array}{cccc}
0 & +\frac{Mx'r'}{r^2}t' + \frac{1}{2}\frac{Mx}{r^3}t' - \frac{Mx}{r^3}t'^3 & +\frac{My'r'}{r^2}t' + \frac{1}{2}\frac{My}{r^3}t' - \frac{My}{r^3}t'^3 & +\frac{Mz'r'}{r^2}t' + \frac{1}{2}\frac{Mz}{r^3}t' - \frac{Mz}{r^3}t'^3 \\
-\frac{Mx'r'}{r^2}t' - \frac{1}{2}\frac{Mx}{r^3}t' + \frac{Mx}{r^3}t'^3 & 0 & \frac{M}{2r^3}(xy'-yx') - \frac{M}{r^3}t'^2(xy'-yx') & \frac{M}{2r^3}(xz'-zx') - \frac{M}{r^3}t'^2(xz'-zx') \\
-\frac{My'r'}{r^2}t' - \frac{1}{2}\frac{My}{r^3}t' + \frac{My}{r^3}t'^3 & -\frac{M}{2r^3}(xy'-yx') + \frac{M}{r^3}t'^2(xy'-yx') & 0 & \frac{M}{2r^3}(yz'-zy') - \frac{M}{r^3}t'^2(yz'-zy') \\
-\frac{Mz'r'}{r^2}t' - \frac{1}{2}\frac{Mz}{r^3}t' + \frac{Mz}{r^3}t'^3 & -\frac{M}{2r^3}(xz'-zx') + \frac{M}{r^3}t'^2(xz'-zx') & -\frac{M}{2r^3}(yz'-zy') + \frac{M}{r^3}t'^2(yz'-zy') & 0
\end{array}
$$

Comparar $\mathcal{A}_{ki}$ con L_{ki}

pág. 18 pág. 15.

$$\mathcal{A}_{23} = +\frac{M}{2r^3}(xy'-yx') - \frac{M}{r^3}t'^2(xy'-yx')$$

$$\mathcal{A}_{23} = -\frac{M}{2r^3}(xy'-yx')\left[2t'^2-1\right]$$

$$\mathcal{A}_{23} = -\frac{M}{2r^3}(xy'-yx')\left[2t'^2-t'^2+x'^2+y'^2+z'^2\right]$$

$$\boxed{\mathcal{A}_{23} = -\frac{M}{2r^3}(xy'-yx')\left[t'^2+x'^2+y'^2+z'^2\right]}$$

$$\mathcal{A}_{12} = +\frac{Mx'r'}{r^2}t' + \frac{1}{2}\frac{Mx}{r^3}t' - \frac{Mx}{r^3}t'^3$$

$$\mathcal{A}_{12} = \frac{Mx't'}{r^3}\left[xx'+yy'+zz'\right] - \frac{Mxt'}{2r^3}\left[2t'^2-1\right]$$

$$\boxed{\mathcal{A}_{12} = \frac{Mx't'}{r^3}\left[xx'+yy'+zz'\right] - \frac{Mxt'}{2r^3}\left[t'^2+x'^2+y'^2+z'^2\right]}$$

Comprobar que

$$\boxed{\mathcal{A}_{ij} v^j = \tfrac{1}{2} a_i}$$

$$\mathcal{A}_{1j} v^j = \left[\frac{M x'^2 r'}{r^2} t' + \frac{1}{2} \frac{M x x'}{r^3} t' - \frac{M x x'}{r^3} t'^3 + \frac{M y'^2 r'}{r^2} t' + \frac{1}{2} \frac{M y y'}{r^3} t' - \frac{M y y'}{r^3} t'^3 + \frac{M z'^2 r'}{r^2} t' + \frac{1}{2} \frac{M z z'}{r^3} t' - \frac{M z z'}{r^3} t'^3 \right]$$

$$\mathcal{A}_{1j} v^j = \left[\frac{M r' t'}{r^2} \left[x'^2 + y'^2 + z'^2 \right] + \frac{1}{2} \frac{M t'}{r^3} \left[x x' + y y' + z z' \right] - \frac{M t'^3}{r^3} \left[x x' + y y' + z z' \right] \right]$$

$$\mathcal{A}_{1j} v^j = \left[\frac{M r' t'}{r^2} \left[t'^2 - 1 \right] + \frac{M r' t'}{2 r^2} \left[1 - 2t'^2 \right] \right]$$

$$A_{1j}\, v^j = \frac{M r' t'}{2r^2}\left[\cancel{2t'^2} - 2 + 1 - \cancel{2t'^2}\right]$$

$$\boxed{A_{1j}\, v^j = -\frac{M r' t'}{2r^2}} \quad = \frac{1}{2} a_1 \qquad \text{Pag. 5}$$

$$A_{3j}\, v^j = \left[\begin{array}{l} -\frac{M y' r'}{r^2} t'^2 - \frac{1}{2}\frac{M y}{r^3} t'^2 + \frac{M y}{r^3} t'^4 \\ -\frac{M}{2r^3} x'(xy' - yx') + \frac{M x'}{r^3}(xy' - yx') t'^2 \\ +\frac{M}{2r^3} z'(yz' - zy') - \frac{M z'}{r^3} t'^2 (yz' - zy') \end{array}\right]$$

$$\cancel{A_{3j}\, v^j = + \frac{M}{2r^3}}$$

$$A_{3j}\, v^j = \left[\begin{array}{l} -\frac{M y'}{r^3}\underbrace{(xx' + yy' + zz')}_{rr'}\underbrace{(1 + x'^2 + y'^2 + z'^2)}_{t'^2} \\ -\frac{1}{2}\frac{M y}{r^3}(1 + x'^2 + y'^2 + z'^2) \\ +\frac{M y}{r^3}\left[\begin{array}{l} 1 + 2x'^2 + 2y'^2 + 2z'^2 + 2x'^2 y'^2 + 2x'^2 z'^2 \\ + 2y'^2 z'^2 + x'^4 + y'^4 + z'^4 \end{array}\right] \\ -\frac{M x'}{2r^3}(xy' - yx')\, 1 \\ +\frac{M x'}{r^3}(xy' - yx')(1 + x'^2 + y'^2 + z'^2) \end{array}\right]$$

$$\left(A_{3j}\, v^j \right)$$

$$\Bigg[-\frac{M y' r'}{r^2} t'^2 \quad - \frac{M x'}{2r^3}(x y' - y x') \quad + \frac{M z'}{2r^3}(y z' - z y')$$

$$- \frac{1}{2}\frac{M y}{r^3} t'^2 \quad + \frac{M x'}{r^3}(x y' - y x') t'^2 \quad - \frac{M z'}{r^3}(y z' - z y') t'^2$$

$$+ \frac{M y}{r^3} t'^4 \Bigg]$$

$$+ \left[\frac{M}{r^3} t'^2 \left\{ \cancel{-x x' y'} - y y'^2 - \cancel{z y' z'} + \cancel{x x' y'} - y x'^2 - y z'^2 + \cancel{z y' z'} \right\} \right]$$

$$+ \left[\frac{M}{2r^3} \left\{ -x x' y' + y x'^2 + y z'^2 - z y' z' \right\} \right]$$

$$+ \left[\frac{M y t'^2}{2 r^3} \left\{ 2t'^2 - 1 \right\} \right]$$

$$A_{3j} v^j$$

$$+\left[-\frac{M t'^2 y}{r^3}\left[x'^2 + y'^2 + z'^2\right]\right]$$

$$+\left[\frac{M}{2r^3}\left\{-x x' y' - z z' y' \underbrace{\cancel{- y y' y'} \cancel{+ y y' y'}} + y x'^2 + y z'^2\right\}\right]$$

$$+\left[\frac{M y t'^2}{2 r^3}\left\{2t'^2 - 1\right\}\right]$$

$$+\left[-\frac{M y t'^2}{r^3}\left\{x'^2 + y'^2 + z'^2\right\}\right]$$

$$+\left[\frac{M}{2r^3}\left\{- r r' y' + y\left(x'^2 + y'^2 + z'^2\right)\right\}\right]$$

$$+\left[\frac{M y}{2 r^3} t'^2 \left\{2t'^2 - 1\right\}\right]$$

$$A_{3j}\,v^j$$

$$+\left[-\frac{My}{r^3}\,t'^2\left(t'^2-1\right)\right]$$

$$+\left[+\frac{Mr'y'}{2r^2}+\frac{My}{2r^3}\left(t'^2-1\right)\right]$$

$$+\left[+\frac{My}{2r^3}\,t'^2\left(2t'^2-1\right)\right]$$

$$A_{3j}\,v^j = +\frac{My}{2r^3}\left[\cancel{-2t'^4}+2t'^2+\cancel{t'^2}-1+\cancel{2t'^4}-\cancel{t'^2}\right]-\frac{Mr'y'}{2r^2}$$

$$A_{3j}\,v^j = \frac{My}{2r^3}\left[2t'^2-1\right]-\frac{Mr'y'}{2r^2}$$

Definición del tensor b_{ij}

$$b_{ij} = \begin{pmatrix} 0 & \frac{Mx'r't'}{r^2} & \frac{My'r't'}{r^2} & \frac{Mz'r't'}{r^2} \\ -\frac{Mx'r't'}{r^2} & 0 & 0 & 0 \\ -\frac{My'r't'}{r^2} & 0 & 0 & \\ -\frac{Mz'r't'}{r^2} & 0 & 0 & 0 \end{pmatrix}$$

$$b_{ij} v^j = \begin{pmatrix} \frac{Mr't'}{r^2}(x'^2 + y'^2 + z'^2) \\ -\frac{Mr't'^2}{r^2} x' \\ -\frac{Mr't'^2}{r^2} y' \\ -\frac{Mr't'^2}{r^2} z' \end{pmatrix}$$

Comprobar que $\{-L_{ki}\, v^k\} = a_i$

$$L_{k1}\, v^k = \left[\frac{Mt'}{r^3}\left(xx' + yy' + zz'\right)\right]$$

$$L_{k1}\, v^k = \frac{Mt'r'}{r^2} = -a_1$$

$$L_{k4}\, v^k = \left[\begin{array}{l} -\dfrac{Mz}{r^3}t'^2 \\ +\dfrac{Mx'}{r^3}\left(-zx' + xz'\right) \\ +\dfrac{My'}{r^3}\left(-zy' + yz'\right) \end{array}\right]$$

$$L_{k4}\, v^k = \left[\begin{array}{l} -\dfrac{Mz}{r^3}t'^2 \\ +\dfrac{M}{r^3}\left[\begin{array}{l} -zx'^2 + xx'z' \\ -zy'^2 + yy'z' \\ -zz'^2 + zz'z' \end{array}\right] \end{array}\right]$$

$$L_{k4} v^k = \begin{bmatrix} -\dfrac{Mz}{r^3} t'^2 \\ -\dfrac{Mz}{r^3}\left(x'^2 + y'^2 + z'^2\right) \\ + \dfrac{Mr'z'}{r^2} \end{bmatrix}$$

$$L_{k4} v^k = -\frac{Mz}{r^3}\left[2t'^2 - 1\right] + \frac{Mr'z'}{r^2}$$

$$\boxed{L_{k4} v^k = \frac{Mz}{r^3} - \frac{2Mz}{r^3} t'^2 + \frac{Mr'z'}{r^2}}$$

Comprobado pág. 9.

$$\boxed{\begin{array}{l} L_{ki} v^k = -a_i \\ \hline L_{ij} v^j = +a_i \end{array}}$$

L antisimétrico.

$$\mathcal{A}_{ij} = b_{ij} + \frac{1}{2} L_{ij} (2t'^2 - 1)$$

$$\mathcal{A}_{ij} v^j = b_{ij} v^j + \frac{1}{2} L_{ij} v^j (2t'^2 - 1)$$

$$\frac{1}{2} a_i = b_{ij} v^j + \frac{1}{2} a_i (2t'^2 - 1)$$

?

CAMPO

SIMBOLOS DE BIR

$$\boxed{\mathcal{B}_{jk,i} = \frac{\partial h_{jk}}{\partial x^i} -}$$

$\mathcal{B}_{11,1}$	0	$\mathcal{B}_{12,1}$	$+\frac{Mx}{r^3}$	$\mathcal{B}_{13,1}$	$+\frac{My}{r^3}$	$\mathcal{B}_{14,1}$	$+\frac{Mz}{r^3}$
$\mathcal{B}_{21,1}$	0	$\mathcal{B}_{22,1}$	-0	$\mathcal{B}_{23,1}$	0	$\mathcal{B}_{24,1}$	0
$\mathcal{B}_{31,1}$	0	$\mathcal{B}_{32,1}$	0	$\mathcal{B}_{33,1}$	0	$\mathcal{B}_{34,1}$	0
$\mathcal{B}_{41,1}$	0	$\mathcal{B}_{42,1}$	0	$\mathcal{B}_{43,1}$	0	$\mathcal{B}_{44,1}$	0
$\mathcal{B}_{11,2}$	$-\frac{Mx}{r^3}$	$\mathcal{B}_{12,2}$	0	$\mathcal{B}_{13,2}$	0	$\mathcal{B}_{14,2}$	0
$\mathcal{B}_{21,2}$	0	$\mathcal{B}_{22,2}$	0	$\mathcal{B}_{23,2}$	$+\frac{My}{r^3}$	$\mathcal{B}_{24,2}$	$+\frac{Mz}{r^3}$
$\mathcal{B}_{31,2}$	0	$\mathcal{B}_{32,2}$	0	$\mathcal{B}_{33,2}$	$-\frac{Mx}{r^3}$	$\mathcal{B}_{34,2}$	0
$\mathcal{B}_{41,2}$	0	$\mathcal{B}_{42,2}$	0	$\mathcal{B}_{43,2}$	0	$\mathcal{B}_{44,2}$	$-\frac{Mx}{r^3}$

CENTRAL

HOFF ASIMETRICOS DE PRIMERA ESPECIE

$\boxed{\frac{\partial h_{ij}}{\partial x^k}}$

$\mathcal{B}_{11,3}$	$-\frac{M_y}{r^3}$	$\mathcal{B}_{12,3}$	0	$\mathcal{B}_{13,3}$	0	$\mathcal{B}_{14,3}$	0
$\mathcal{B}_{21,3}$	0	$\mathcal{B}_{22,3}$	$-\frac{M_y}{r^3}$	$\mathcal{B}_{23,3}$	0	$\mathcal{B}_{24,3}$	0
$\mathcal{B}_{31,3}$	0	$\mathcal{B}_{32,3}$	$+\frac{M_x}{r^3}$	$\mathcal{B}_{33,3}$	0	$\mathcal{B}_{34,3}$	$+\frac{M_z}{r^3}$
$\mathcal{B}_{41,3}$	0	$\mathcal{B}_{42,3}$	0	$\mathcal{B}_{43,3}$	0	$\mathcal{B}_{44,3}$	$-\frac{M_y}{r^3}$
$\mathcal{B}_{11,4}$	$-\frac{M_z}{r^3}$	$\mathcal{B}_{12,4}$	0	$\mathcal{B}_{13,4}$	0	$\mathcal{B}_{14,4}$	0
$\mathcal{B}_{21,4}$	0	$\mathcal{B}_{22,4}$	$-\frac{M_z}{r^3}$	$\mathcal{B}_{23,4}$	0	$\mathcal{B}_{24,4}$	0
$\mathcal{B}_{31,4}$	0	$\mathcal{B}_{32,4}$	0	$\mathcal{B}_{33,4}$	$-\frac{M_z}{r^3}$	$\mathcal{B}_{34,4}$	0
$\mathcal{B}_{41,4}$	0	$\mathcal{B}_{42,4}$	$+\frac{M_x}{r^3}$	$\mathcal{B}_{43,4}$	$+\frac{M_y}{r^3}$	$\mathcal{B}_{44,4}$	0

Comprobado en el cuaderno 33 página 74

Comprobado pág 15.

$$\boxed{L_{ki} = \mathcal{B}_{jk,i}\, v^j}$$

Ver página 15.

Antisimétrico

0	$-\frac{Mx}{r^3}t'$	$-\frac{My}{r^3}t'$	$-\frac{Mz}{r^3}t'$
$+\frac{Mx}{r^3}t'$	0	$\frac{M}{r^3}(-yx'+xy')$	$\frac{M}{r^3}(-zx'+xz')$
$+\frac{My}{r^3}t'$	$\frac{M}{r^3}(yx'-xy')$	0	$\frac{M}{r^3}(-zy'+yz')$
$+\frac{Mz}{r^3}t'$	$\frac{M}{r^3}(zx'-xz')$	$\frac{M}{r^3}(zy'-yz')$	0

Comparar L_{ki} con $\mathcal{A}(D_{ki})$
pág. 29 página 8.

$$\mathcal{A}(D_{ki}) = \frac{3}{4} L_{ki}$$

Comprobar que $L_{ki} \, v^i = a_k$

$$\boxed{L_{ki} \, v^i = a_k}$$

$$a_1 = -\frac{M x x'}{r^3} t' - \frac{M y y'}{r^3} t' - \frac{M z z'}{r^3} t'$$

$$a_1 = -\frac{M r r'}{r^3} t' = -\frac{M r' t'}{r^2}$$

$$\boxed{a_1 = -\frac{M r' t'}{r^2}}$$

Comprobado pág. 5.

$$a_2 = \left\{ + \frac{M x}{r^3} t'^2 + \frac{M}{r^3} y'(-y x' + x y') + \frac{M}{r^3} z'(-z x' + x z') \right\}$$

$$a_2 = \frac{M x}{r^3} t'^2 + \frac{M}{r^3} \left[-y x' y' + x y'^2 - z x' z' + x z'^2 \right]$$

$$a_2 = \frac{M x}{r^3} t'^2 + \frac{M}{r^3} \left[-x x' x' - y x' y' - z x' z' + x x'^2 + x y'^2 + x z'^2 \right]$$

$$a_2 = \frac{Mx}{r^3} t'^2 + \frac{M}{r^3}\left[-r r' x' + x\{x'^2 + y'^2 + z'^2\}\right]$$

$$a_2 = \frac{Mx}{r^3} t'^2 \cancel{+ \frac{M}{r^3}}\left[-\frac{Mx'r'}{r^2} + \frac{Mx}{r^3}(x'^2 + y'^2 + z'^2)\right)$$

$$a_2 = \frac{Mx}{r^3} t'^2 + \frac{Mx}{r^3}(x'^2 + y'^2 + z'^2) - \frac{Mx'r'}{r^2}$$

$$\boxed{a_2 = \frac{Mx}{r^3} + \frac{2Mx}{r^3}(x'^2 + y'^2 + z'^2) - \frac{Mx'r'}{r^2}}$$

pág. 5.

$$\boxed{L_{ki} v^i = a_k}$$

$$\boxed{L_{ki} v^k = -a_i}$$

$$\boxed{M_{ki} = \mathcal{B}_{kji}\, w^{i}}$$

Comprobado pag. 16

$$
\begin{array}{c|c|c|c}
\dfrac{Mr'}{r^2} & -\dfrac{Mx}{r^3}t' & -\dfrac{My}{r^3}t' & -\dfrac{Mz}{r^3}t' \\
\hline
0 & \dfrac{M}{r^3}(yy'+zz') & -\dfrac{Myx'}{r^3} & -\dfrac{Mzx'}{r^3} \\
\hline
0 & -\dfrac{Mxy'}{r^3} & \dfrac{M}{r^3}(xx'+zz') & -\dfrac{Mzy'}{r^3} \\
\hline
0 & -\dfrac{Mxz'}{r^3} & -\dfrac{Myz'}{r^3} & \dfrac{M}{r^3}(xx'+yy')
\end{array}
$$

Comprobar que

$$\boxed{M_{ki} v^k = - a_i}$$

Ver página 13.

$$\boxed{- a_1 = \frac{M r' t'}{r^2}}$$

pág. 5.

$$\cancel{- a_2 = - \frac{M t'}{r^3} (x x'^2 + y y' + z z')}$$

$$- a_2 = \left\{ \begin{array}{l} - \frac{M x}{r^3} t'^2 + \frac{M}{r^3} x' (y y' + z z') \\ - \frac{M}{r^3} x y'^2 - \frac{M}{r^3} x z'^2 \end{array} \right\}$$

$$- a_2 = - \frac{M x}{r^3} t'^2 + \frac{M}{r^3} \left[\begin{array}{l} y x' y' + z x' z' + x x' x' \\ - x y'^2 - x z'^2 - x x'^2 \end{array} \right.$$

$$- a_2 = - \frac{M x}{r^3} t'^2 + \frac{M x'}{r^3} (r r') - \frac{M x}{r^3} (x'^2 + y'^2 + z'^2)$$

$$a_2 = \frac{M x}{r^3} [1 + x'^2 + y'^2 + z'^2] - \frac{M x' r'}{r^2} + \frac{M x}{r^3} (x'^2 + y'^2 + z'^2)$$

$$\boxed{a_2 = \frac{M x}{r^3} + \frac{2 M x}{r^3} (x'^2 + y'^2 + z'^2) - \frac{M x' r'}{r^2}}$$

pág. 5

Parte antisimétrica de M_{ki}

$$\mathcal{A}(M_{ki}) = \frac{1}{2} L_{ki}$$

$$\boxed{\mathcal{A}(M_{ki}) = \frac{1}{2}(M_{ki} - M_{ik})}$$

0	$-\frac{1}{2}\frac{Mx}{r^3}t'$	$-\frac{1}{2}\frac{My}{r^3}t'$	$-\frac{1}{2}\frac{Mz}{r^3}t'$
$+\frac{1}{2}\frac{Mx}{r^3}t'$	0	$+\frac{M}{2r^3}(xy'-yx')$	$+\frac{M}{2r^3}(xz'-zx')$
$+\frac{1}{2}\frac{My}{r^3}t'$	$+\frac{M}{2r^3}(yx'-xy')$	0	$+\frac{M}{2r^3}(yz'-zy')$
$+\frac{1}{2}\frac{Mz}{r^3}t'$	$+\frac{M}{2r^3}(zx'-xz'$	$+\frac{M}{2r^3}(zy'-yz')$	0

$$\boxed{\mathcal{A}(M_{ki}) = \tfrac{1}{2} L_{ki}}$$

pág. 34 pág. 29

$$\boxed{\mathcal{S}(M_{ki}) = \tfrac{1}{2}(M_{ki} + M_{ik})}$$

Parte simétrica de M_{ki}

$\boxed{\mathcal{S}_{ki}}$

$$\boxed{\mathcal{S}(M_{ki}) = \frac{1}{2}(M_{ki} + M_{ik})}$$

$\frac{Mr'}{r^2}$	$-\frac{Mx}{2r^3}t'$	$-\frac{My}{2r^3}t'$	$-\frac{Mz}{2r^3}t'$
$-\frac{Mx}{2r^3}t'$	$\frac{M}{r^3}(yy'+zz')$	$-\frac{M}{2r^3}(xy'+yx')$	$-\frac{M}{2r^3}(xz'+zx')$
$-\frac{My}{2r^3}t'$	$-\frac{M}{2r^3}(xy'+yx')$	$\frac{M}{r^3}(xx'+zz')$	$-\frac{M}{2r^3}(yz'+zy')$
$-\frac{Mz}{2r^3}t'$	$-\frac{M}{2r^3}(xz'+zx')$	$-\frac{M}{2r^3}(yz'+zy')$	$\frac{M}{r^3}(xx'+yy')$

$$\mathcal{S}_{ki}\, v^i$$

$$\mathcal{S}_{ki} v^i = \frac{M r' t'}{r^2} - \frac{M x x'}{2r^3} t' - \frac{M y y'}{2r^3} t' - \frac{M z z'}{2r^3} t'$$

$$\mathcal{S}_{1i}\, v^i = \frac{M r' t'}{r^2} - \frac{M r' t'}{2r^2} = \frac{1}{2} \frac{M r' t'}{r^2}$$

$$\boxed{\mathcal{S}_{1i}\, v^i = \frac{1}{2} \frac{M r' t'}{r^2}}$$

$$\mathcal{S}_{2i}\, v^i = -\frac{M x}{2r^3} t'^2 + \frac{M}{2r^3} \left[\begin{array}{l} 2y x' y' + 2z x' z' \\ - x y'^2 - \cancel{y} x' y' \\ - x z'^2 - z x' z' \end{array} \right]$$

$$\cancel{\mathcal{S}_{2i}\, v^i = -\frac{M x}{2r^3} t'^2 + \frac{M x}{r^3} \left[-x'^2 - x^2 - z^2 \right] - \frac{M x x'^2}{r^3}}$$

$$\mathcal{S}_{2i}\, v^i = -\frac{M x}{2r^3} t'^2 + \frac{M}{2r^3} \left[\begin{array}{l} x x' x' + y y' x' + z z' x' \\ - x x'^2 - x y'^2 - x z'^2 \end{array} \right]$$

$$\mathcal{S}_{2i} v^i = -\frac{Mx}{2r^3} t'^2 + \frac{M r' x'}{2r^2} - \frac{Mx}{2r^3}(x'^2 + y'^2 + z'^2)$$

$$\boxed{\mathcal{S}_{2i} v^i = -\frac{Mx}{2r^3} - \frac{Mx}{r^3}(x'^2 + y'^2 + z'^2) + \frac{M r' x'}{2r^2}}$$

$$\boxed{\mathcal{S}_{ki} v^i = -\tfrac{1}{2} a_k}$$ pág. 5.

Hay un tensor simétrico $-2\mathcal{S}_{ki}$ y un tensor antisimétrico L_{ki} que transforman al cuadrivector v^i en a_i:

$$(L_{ki} + 2\mathcal{S}_{ki}) v^i \overset{?}{=} 0$$

$$\boxed{\mathcal{O}_{ki} = L_{ki} + 2\mathcal{S}_{ki}}$$

$$\boxed{\Theta_{ki} = \underline{\underline{2M_{ki}}}}$$ ← pág. 32

$$\boxed{\Theta_{ki} = \underset{\text{pág. 29}}{L_{ki}} + 2\underset{\text{pág. 3}}{S_{ki}}}$$ págs 27 y 36

$$\boxed{\mathcal{O}_{ki} = \underset{\text{pág. 29}}{L_{ki}} + 2\underset{\text{pág. 36}}{S_{ki}}}$$ págs 21, 7 — 36

$+\frac{2Mr'}{r^2}$	$-\frac{2Mx}{r^3}t'$	$-\frac{2My}{r^3}t'$	$-\frac{2Mz}{r^3}t'$
0	$+\frac{2M}{r^3}(yy'+zz')$	$-\frac{2My}{r^3}x'$	$-\frac{2Mz}{r^3}x'$
0	$-\frac{2Mx}{r^3}y'$	$+\frac{2M}{r^3}(xx'+zz')$	$-\frac{2Mz}{r^3}y'$
0	$-\frac{2M}{r^3}xz'$	$-\frac{2My}{r^3}z'$	$+\frac{2M}{r^3}(xx'+yy')$

$$\mathcal{O}_{ki}\, v^i =$$

$$\frac{2M}{r^3}$$

$$\mathcal{O}_{1i}\, v^i = \frac{2M}{r^3}\left[r r' - x x' - y y' - z z' \right] t'$$

$$\mathcal{O}_{2i}\, v^i = \frac{2M}{r^3}\left[y x' y' + z x' z' - y x' y' - z x' z' \right]$$

$$\mathcal{O}_{3i}\, v^i = \frac{2M}{r^3}\left[- x x' y' + x x' y' + z y' z' - z y' z' \right]$$

$$\mathcal{O}_{4i}\, v^i = + \frac{2M}{r^3}\left[- x x' z' - y y' z' + x x' z' + y y' z' \right]$$

$$\boxed{\mathcal{O}_{ki}\, v^i = \underset{\text{cuadrivector nulo}}{0^{1}}}$$

Tabla de Tensores.

Símbolo	Nombre	Definición	Efecto sobre el cuadrivector velocidad.
$\mathcal{B}_{jk,i}$	Símbolos de Birkhoff de primera especie, asimétricos (en j y k)	$\dfrac{\partial h_{jk}}{\partial x^i} - \dfrac{\partial h_{ij}}{\partial x^k}$ cuaderno 33 págs. 0', 73, 74	$\mathcal{B}_{jk,i} v^j = L_{ki}$ asimétrico pág 29 $\mathcal{B}_{jk,i} v^k = M_{ji}$ pág 52
$B_{jk,i}$	Símbolos de Birkhoff de primera especie simétricos (en j y k)	$\dfrac{\partial h_{jk}}{\partial x^i} - \dfrac{1}{2}\dfrac{\partial h_{ij}}{\partial x^k} - \dfrac{1}{2}\dfrac{\partial h_{ik}}{\partial x^j}$ cuaderno 33 pág. 1.	$B_{jk,i} v^j = D_{ki}$ pág 3.
L_{ki}	Tensor intermedio antisimétrico	$\mathcal{B}_{jk,i} v^j = L_{ki}$ pág. 29	$L_{ki} v^k = -a_i$ pág 26 $L_{ki} v^i = +a_k$ pág 26

M_{ji}	Segundo tensor intermedio	$\mathcal{B}_{jk,i}\, v^k = M_{ji}$ pág 32	$M_{ji}\, v^j = -a_i$ pág 3 $M_{ji}\, v^i = \vec{0}$ pág. 42
D_{ki}		$B_{jk,i}\, v^j = D_{ki}$	$D_{ki}\, v^k = -a_i$ pág. 44 $D_{ki}\, v^i = \frac{1}{2} a_k$ pág. 45
			$\mathcal{B}_{jk,i}\, v^i = -M_{jk}$ pág. 4 $B_{jk,i}\, v^i = S_{jk} = S(M_{jk})$
$S_{ji} = S(M_{ji})$			$S_{ji}\, v^i = -\frac{1}{2} a_j$ pág. 52

$$\boxed{M_{ki}\, v^i}$$

$$M_{1i} v^i = \frac{M r' t'}{r^2} - \frac{M x x'}{r^3} t' - \frac{M y y'}{r^3} t' - \frac{M z z'}{r^3} t'$$

$$M_{1i} v^i = \frac{M r' t'}{r^2} - \frac{M r r'}{r^2} t' = 0$$

$$M_{2i} v^i = \frac{M}{r^3}\left[\cancel{y x' y'} + \cancel{z x' z'} - \cancel{y x' y'} - \cancel{z x' z'}\right]$$

$$M_{2i} v^i = 0$$

$$\boxed{M_{ki} v^i = 0}$$

$$D_{k1} v^k = \frac{M}{2r^3}\left[r r' t' + x x' t' + y y' t' + z z' t'\right]$$

$$\boxed{D_{k1} v^k = \cancel{\frac{M r t'}{r^2}}\ \frac{M r' t'}{r^2} = -a_1}$$ pág. 5

$$D_{k2} v^k = -\frac{M x t'^2}{r^3} + \frac{M}{2r^3}\left[\begin{array}{l} r r' x' - x x'^2 \\ + y y' x' - 2\cancel{x} y'^2 \\ + z x' z' - 2 x z'^2 \end{array}\right]$$

$$D_{k2} v^k = -\frac{M x t'^2}{r^3} + \frac{M}{2r^3}\left[\begin{array}{l} r r' x' + x x'^2 \\ x' r r' - 2x\left\{\begin{array}{l} x'^2 + y'^2 \\ + z'^2 \end{array}\right\} \end{array}\right]$$

$$D_{k2} v^k = -\frac{Mt'^2}{r^3} x + \frac{M}{2r^3}\left[rr'x' - xx'^2 + yy'x' - 2xy'^2 + zz'x' - 2xz'^2 \right]$$

$$D_{k2} v^k = \frac{M}{2r^3}\left[-2x\left(1 + x'^2 + y'^2 + z'^2\right) + \left(xx' + yy' + zz'\right)x' - xx'^2 - 2xy'^2 - 2xz'^2 + yy'x' + zz'x' \right]$$

$$D_{k2} v^k = \frac{M}{2r^3}\left[\begin{array}{l} -2x - 2xx'^2 - 2xy'^2 - 2xz'^2 \\ \quad + xx'^2 \\ \quad - xx'^2 - 2xy'^2 - 2xz'^2 \\ + 2xx'x' \quad - 2xx'^2 \\ + yy'x' + zz'x' \\ + yy'x' + zz'x' \end{array} \right]$$

$$D_{k2} v^k = \frac{M}{2r^3}\left[-2x - 4x\left(x'^2 + y'^2 + z'^2\right) + 2x'rr' \right]$$

$$D_{k2} v^k = -\frac{Mx}{r^3} - \frac{2Mx}{r^3}\left(x'^2 + y'^2 + z'^2\right) + \frac{Mx'r'}{r^2}$$

$$\boxed{D_{k2} v^k = -a_2}$$

página 5.

$$\boxed{D_{ki}\, v^k = -a_i}$$ pág. 5.

$$D_{1i}\, v^i = \frac{M}{2r^3} r r' t' - \frac{M t'}{r^3}(x x' + y y' + z z')$$

$$D_{1i}\, v^i = \frac{M}{2r^2} r' t' - \frac{M r' t'}{r^2} = -\frac{M}{2r^2} r' t'$$

$$\boxed{D_{1i}\, v^i = -\frac{M}{2r^2} r' t'} \quad \boxed{D_{1i}\, v^i = \tfrac{1}{2} a_1}$$ pág. 5

$$D_{4i}\, v^i = \frac{Mz}{2r^3} t'^2 + \frac{M}{2r^3}\left[z x'^2 - 2 x x' z' + z y'^2 - 2 y y' z' + x x' z' + y y' z' + z z'^2 - z z'^2 \right]$$

$$D_{4i}\, v^i = \frac{M}{2r^3}\left[z + z x'^2 + z y'^2 + z z'^2 + z x'^2 + z y'^2 + z z'^2 - x x' z' - y y' z' - z z' z' \right]$$

$$D_{4i}\, v^i = \frac{M}{2r^3}\left[z + 2z(x'^2 + y'^2 + z'^2) - r r' z' \right]$$

$$\boxed{D_{4i}\, v^i = \frac{Mz}{2r^3} + \frac{Mz}{r^3}(x'^2 + y'^2 + z'^2) - \frac{M r' z'}{2r^2}}$$

$$\boxed{D_{4i}\, v^i = \tfrac{1}{2} a_4}$$ pág. 5.

$$\boxed{D_{ki}\, v^i = \tfrac{1}{2} a_k}$$ pág. 5.

$$\mathcal{B}_{jk,i}\, v^i = \,?$$

$$\mathcal{B}_{11,i}\, v^i = -\frac{M}{r^3}\big[xx' + yy' + zz'\big]$$

$$\boxed{\mathcal{B}_{11,i}\, v^i = -\frac{M r'}{r^2}}$$

$$\boxed{\mathcal{B}_{12,i}\, v^i = \frac{Mx}{r^3}\, t'}$$

$$\boxed{\mathcal{B}_{13,i}\, v^i = \frac{My}{r^3}\, t'}$$

$$\boxed{\mathcal{B}_{14,i}\, v^i = \frac{Mz}{r^3}\, t'}$$

$$\mathcal{B}_{21,i}\, v^i = 0$$

$$\mathcal{B}_{22,i}\, v^i = -\frac{M}{r^3}(yy' + zz')$$

$$\mathcal{B}_{23,i}\, v^i = +\frac{M}{r^3}(yx')$$

$$\mathcal{B}_{24,i}\, v^i = +\frac{M}{r^3}(zx')$$

$$\mathcal{B}_{31,i} v^i = 0$$

$$\mathcal{B}_{32,i} v^i = \frac{Mx}{r^3} y'$$

$$\mathcal{B}_{33,i} v^i = -\frac{M}{r^3}(xx' + zz')$$

$$\mathcal{B}_{34,i} v^i = +\frac{Mz}{r^3} y'$$

$$\mathcal{B}_{41,i} v^i = 0$$

$$\mathcal{B}_{42,i} v^i = +\frac{Mx}{r^3} z'$$

$$\mathcal{B}_{43,i} v^i = +\frac{My}{r^3} z'$$

$$\mathcal{B}_{44,i} v^i = -\frac{M}{r^3}(xx' + yy')$$

$$\boxed{\mathcal{B}_{jk,i}\, v^i = - M_{jk}}$$ pág. 16

$-\frac{Mr'}{r^2}$	$+\frac{Mx'}{r^3}t'$	$+\frac{My'}{r^3}t'$	$+\frac{Mz'}{r^3}t'$
0	$-\frac{M}{r^3}(yy'+zz')$	$+\frac{My}{r^3}x'$	$+\frac{Mz}{r^3}x'$
0	$+\frac{Mx}{r^3}y'$	$-\frac{M}{r^3}(xx'+zz')$	$+\frac{Mz}{r^3}y'$
0	$+\frac{Mx}{r^3}z'$	$+\frac{My}{r^3}z'$	$-\frac{M}{r^3}(xx'+yy')$

CAMPO

SIMBOLOS DE BIRK

$$B_{jk,i} = \frac{\partial h_{jk}}{\partial x^i} -$$

$B_{11,1}$	0	$B_{12,1}$	$+\frac{Mx}{2r^3}$	$B_{13,1}$	$+\frac{My}{2r^3}$	$B_{14,1}$	$+\frac{Mz}{2r^3}$
$B_{21,1}$	$+\frac{Mx}{2r^3}$	$B_{22,1}$	0	$B_{23,1}$	0	$B_{24,1}$	0
$B_{31,1}$	$+\frac{My}{2r^3}$	$B_{32,1}$	0	$B_{33,1}$	0	$B_{34,1}$	0
$B_{41,1}$	$+\frac{Mz}{2r^3}$	$B_{42,1}$	0	$B_{43,1}$	0	$B_{44,1}$	0
$B_{11,2}$	$-\frac{Mx}{r^3}$	$B_{12,2}$	0	$B_{13,2}$	0	$B_{14,2}$	0
$B_{21,2}$	0	$B_{22,2}$	0	$B_{23,2}$	$+\frac{My}{2r^3}$	$B_{24,2}$	$+\frac{Mz}{2r^3}$
$B_{31,2}$	0	$B_{32,2}$	$+\frac{My}{2r^3}$	$B_{33,2}$	$-\frac{Mx}{r^3}$	$B_{34,2}$	0
$B_{41,2}$	0	$B_{42,2}$	$+\frac{Mz}{2r^3}$	$B_{43,2}$	0	$B_{44,2}$	$-\frac{Mx}{r^3}$

CENTRAL

L. HOFF DE PRIMERA ESPECIE, SIMETRICOS

$$\boxed{\frac{1}{2}\frac{\partial h_{ij}}{\partial x^k} - \frac{1}{2}\frac{\partial h_{ik}}{\partial x^j}}$$

$B_{11,3}$	$-\frac{My}{r^3}$	$B_{12,3}$	0	$B_{13,3}$	0	$B_{14,3}$	0
$B_{21,3}$	0	$B_{22,3}$	$-\frac{My}{r^3}$	$B_{23,3}$	$+\frac{Mx}{2r^3}$	$B_{24,3}$	0
$B_{31,3}$	0	$B_{32,3}$	$+\frac{Mx}{2r^3}$	$B_{33,3}$	0	$B_{34,3}$	$+\frac{Mz}{2r^3}$
$B_{41,3}$	0	$B_{42,3}$	0	$B_{43,3}$	$+\frac{Mz}{2r^3}$	$B_{44,3}$	$-\frac{My}{r^3}$
$B_{11,4}$	$-\frac{Mz}{r^3}$	$B_{12,4}$	0	$B_{13,4}$	0	$B_{14,4}$	0
$B_{21,4}$	0	$B_{22,4}$	$-\frac{Mz}{r^3}$	$B_{23,4}$	0	$B_{24,4}$	$+\frac{Mx}{2r^3}$
$B_{31,4}$	0	$B_{32,4}$	0	$B_{33,4}$	$-\frac{Mz}{r^3}$	$B_{34,4}$	$+\frac{My}{2r^3}$
$B_{41,4}$	0	$B_{42,4}$	$+\frac{Mx}{2r^3}$	$B_{43,4}$	$+\frac{My}{2r^3}$	$B_{44,4}$	0

$$\boxed{B_{jk,i}\, v^i = -\mathcal{S}(M_{jk}) = \mathcal{S}_{jk} \quad \text{pág. } 36}$$

$-\frac{Mr'}{r^2}$	$+\frac{Mx}{2r^3}t'$	$+\frac{My}{2r^3}t'$	$+\frac{Mz}{2r^3}t'$
$+\frac{Mx}{2r^3}t'$	$-\frac{M}{r^3}(yy'+zz')$	$+\frac{M}{2r^3}(yx'+xy')$	$+\frac{M}{2r^3}(zx'+xz')$
$+\frac{My}{2r^3}t'$	$+\frac{M}{2r^3}(yx'+xy')$	$-\frac{M}{r^3}(xx'+zz')$	$+\frac{M}{2r^3}(zy'+yz')$
$+\frac{Mz}{2r^3}t'$	$+\frac{M}{2r^3}(zx'+xz')$	$+\frac{M}{2r^3}(zy'+yz')$	$-\frac{M}{r^3}(xx'+yy')$

$$\mathcal{S}(D_{jk}) = -\frac{1}{2}\mathcal{S}(M_{jk}) = -\frac{1}{2}S_{jk}.$$

$$\mathcal{A}(D_{jk}) = \frac{3}{4}L_{jk}$$

$$\boxed{D_{jk} = +\frac{1}{2}S_{jk} + \frac{3}{4}L_{jk}}$$

$$M_{jk} = S_{jk} + \mathcal{A}(M_{jk})$$

$$\boxed{M_{jk} = S_{jk} + \frac{1}{2}L_{jk}}$$ pág. 35

$$\mathcal{S}_{1i} v^i = \frac{M r' t'}{r^2} - \frac{M t'}{2r^3}(x x' + y y' + z z')$$

$$\mathcal{S}_{1i} v^i = \frac{M r' t'}{r^2} - \frac{M r r' t'}{2r^3}$$

$$\boxed{\boxed{\mathcal{S}_{1i} v^i = \frac{M r' t'}{2r^2}} \boxed{= -\frac{1}{2} a_1}}$$ pág. 5

$$\mathcal{S}_{4i} v^i = -\frac{M z t'^2}{2r^3} - \frac{M}{2r^3}\left[\begin{matrix} x x' z' + z x'^2 \\ + y y' z' + z y'^2 \\ - 2 x x' z' - 2 y y' z' \end{matrix}\right]$$

$$\cancel{\mathcal{S}_{4i} v^i = -\frac{M}{2r^3}\left[z x'^2 + z y'^2 + z \right.}$$

$$\cancel{\mathcal{S}_{4i} v^i = \mp \frac{M z}{2r^3}}$$

$$\mathcal{S}_{4i} v^i = \frac{M z}{2r^3} - \frac{M}{2r^3}\left[\begin{matrix} z x'^2 + z y'^2 + z z'^2 \\ + z x'^2 + z y'^2 + z z'^2 \\ - x x' z' \quad - z z' z' \\ - y y' z' \end{matrix}\right]$$

$$\delta_{4i}v^i = -\frac{Mz}{2r^3} - \frac{M}{2r^3}\left[\begin{array}{l}2z(x'^2+y'^2+z'^2)\\ -rr'z'\end{array}\right]$$

$$\boxed{\delta_{4i}v^i = -\frac{Mz}{2r^3} - \frac{Mz}{r^3}(x'^2+y'^2+z'^2) + \frac{Mr'z'}{2r^2}}$$

$$\boxed{\delta_{4i}v^i = -\frac{1}{2}a_4}$$ pág. 5

$$\boxed{\delta_{ji}v^i = -\frac{1}{2}a_j}$$

$$M_{jk} = \delta_{jk} + \frac{1}{2}L_{jk}$$ pág. 50

$$\cancel{M_{jk}v^k = \delta_{jk}v^j + \frac{1}{2}L_{jk}v^j}$$

$$\cancel{M_{jk}v^j =}$$

$$M_{jk}v^k = \delta_{jk}v^k + \frac{1}{2}L_{jk}v^k$$

pág. 26

$$M_{jk}v^k = -\frac{1}{2}a_j + \frac{1}{2}a_j = 0$$

$$\cancel{M_{jk}v^k = \frac{1}{2}a_j}$$

Para construir una Tabla de Tensores.

Tercer orden.

1) $\mathcal{B}_{jk,i}$ símbolos asimétricos de Birkhoff de primera especie;

2) $B_{jk,i}$ símbolos simétricos de Birkhoff de primera especie;

3) $\mathcal{B}_{jk,i}\, v^j = L_{ki}$ ← tensor antisimétrico

4) $\mathcal{B}_{jk,i}\, v^k = M_{ji}$

5) $\mathcal{S}_{ji} = \mathcal{S}(M_{ji}) = \frac{1}{2}(M_{ji} + M_{ij})$

$\mathcal{A}(M_{ji}) = \frac{1}{2}(M_{ji} - M_{ij}) = \frac{1}{2} L_{ji}$

6) $M_{ji} = \mathcal{S}_{ji} + \frac{1}{2} L_{ji}$

Investíguese el efecto de todos estos tensores sobre el cuadrivector velocidad.

$$\mathcal{B}_{jk,i}\, v^j = L_{ki}$$

$$\mathcal{B}_{jk,i}\, v^k = \mathcal{S}_{ji} + \tfrac{1}{2} L_{ji} = M_{ji}$$

$$\mathcal{B}_{jk,i}\, v^i = \mathcal{S}_{jk}$$

$\mathcal{A}$ parte antisimé.

$\mathcal{S}$ parte simétrica

$$B_{jk,i}\, v^j = + \tfrac{1}{2} \mathcal{S}_{ki} + \tfrac{3}{4} L_{ki}$$

$$B_{jk,i}\, v^k = + \tfrac{1}{2} \mathcal{S}_{ji} + \tfrac{3}{4} L_{ji}$$

$$B_{jk,i}\, v^i = \mathcal{S}_{jk}$$

$$\mathcal{S}_{ij}\, v^i = -\tfrac{1}{2} a_j \qquad M_{ij}\, v^i = -a_j$$ pág. 33

$$\mathcal{S}_{ij}\, v^j = -\tfrac{1}{2} a_i \qquad M_{ij} \cdot v^j = 0$$ pág 42

$$L_{ij}\, v^i = -a_j$$

$$L_{ij}\, v^j = + a_i$$

Calcular la parte simétrica y la parte antisimétrica del tensor $\alpha_{ij} = a_i v_j$ pág. 11.

pág. 18. $\mathcal{A}(\alpha_{ij}) = \frac{1}{2}(\alpha_{ij} - \beta_{ij}) = \frac{1}{2}(\alpha_{ij} - \alpha_{ji})$

$$\boxed{\beta_{ij} = + \alpha_{ji}}$$

$$\mathcal{S}(\alpha_{ij}) = \frac{1}{2}(\alpha_{ij} + \beta_{ij}) = \frac{1}{2}(\alpha_{ij} + \alpha_{ji}) .$$

Comprobar: $\alpha_{ij} v^j = a_i$

$$\alpha_{1j} v^j = -\frac{M r'}{r^2} t'^3 + \frac{M r' t'}{r^2} x'^2 + \frac{M r' t'}{r^2} y'^2 + \frac{M r' t'}{r^2} z'^2$$

$$\alpha_{1j} v^j = \frac{M r' t'}{r^2}\left[-t'^2 + x'^2 + y'^2 + z'^2\right]$$

$$\boxed{\alpha_{1j} v^j = -\frac{M r' t'}{r^2}}$$ Ver página 9.

$$\alpha_{3j} v^j = \begin{cases} -\frac{M_y}{r^3} t'^2 + \frac{M_y x'^2}{r^3} + \frac{M_y y'^2}{r^3} + \frac{M_y z'^2}{r^3} \\ + \frac{2 M_y}{r^3} t'^4 - \frac{2 M_y x'^2}{r^3} t'^2 - \frac{2 M_y y'^2}{r^3} t'^2 - \frac{2 M_y z'^2}{r^3} t'^2 \\ - \frac{M y' r'}{r^2} t'^2 + \frac{M y' x'^2 r'}{r^2} + \frac{M y'^3 r'}{r^2} + \frac{M y' z'^2 r'}{r^2} \end{cases}$$

$$\alpha_{3j} v^j = \begin{cases} \frac{M_y}{r^3}\left[-t'^2 + x'^2 + y'^2 + z'^2\right] \\ \frac{2 M_y}{r^3} t'^2 \left[t'^2 - x'^2 - y'^2 - z'^2\right] \\ + \frac{M y' r'}{r^2}\left[-t'^2 + x'^2 + y'^2 + z'^2\right] \end{cases}$$

$$\boxed{\alpha_{3j} v^j = -\frac{M_y}{r^3} + \frac{2 M_y}{r^3} t'^2 - \frac{M y' r'}{r^2}}$$ Ver página 9

$$\boxed{\alpha_{ij} v^j = a_i}$$ ¡Comprobado!

Comprobar que $\alpha_{ij} v^i = \vec{0}$.

$$\alpha_{ij} = a_i v_j$$

$$\alpha_{ij} v^i = a_i v_j v^i = 0$$

$$\alpha_{i1} v^i = \left[-\frac{Mr'}{r^2} t'^3 - \frac{Mxx'}{r^3} t' + \frac{2Mxx'}{r^3} t'^3 - \frac{Mx'^2 r'}{r^2} t' - \frac{Myy'}{r^3} t' + \frac{2Myy'}{r^3} t'^3 - \frac{My'^2 r'}{r^2} t' - \frac{Mzz'}{r^3} t' + \frac{2Mzz'}{r^3} t'^3 - \frac{Mz'^2 r'}{r^2} t' \right]$$

$$\alpha_{i1} v^i = \left[-\frac{Mr'}{r^2} t'^3 - \frac{Mr't'}{r^2} + \frac{2Mr'}{r^2} t'^3 - \frac{Mr't'}{r^2} \{ x'^2 + y'^2 + z'^2 \} \right]$$

$$\alpha_{i1} v^i = \frac{Mr'}{r^2} t'^3 - \frac{Mr't'}{r^2} \{ 1 + x'^2 + y'^2 + z'^2 \}$$

$$\alpha_{i1} v^i = \frac{Mr'}{r^2} t'^3 - \frac{Mr't'^3}{r^2} = 0$$

$$\boxed{\alpha_{i1} v^i = 0}$$

$$\alpha_{i2}\, v^i = \frac{M r' x'}{r^2}\left[t'^2 + 1 - 2t'^2 + x'^2 + y'^2 + z'^2 \right]$$

$$\alpha_{i2}\, v^i = \frac{M r' x'}{r^2} \cdot \left[-t'^2 + x'^2 + y'^2 + z'^2 + 1 \right]$$

$$\boxed{\alpha_{i2}\, v^i = 0}$$

$$\boxed{\alpha_{ij}\, v^i = \hat{0}}$$ Comprobado.

$$\mathcal{S}(\alpha_{ij}) = \Sigma_{ij}$$

Parte simétrica de α_{ij} (pág 11)

$$\boxed{\mathcal{S}(\alpha_{ij})} = \Sigma_{ij}$$

$-\frac{Mr'}{r^2}t'^2$	$-\frac{Mx}{2r^3}t'$ $+\frac{Mx}{r^3}t'^3$	$-\frac{My}{2r^3}t'$ $+\frac{My}{r^3}t'^3$	$-\frac{Mz}{2r^3}t'$ $+\frac{Mz}{r^3}t'^3$
$-\frac{Mx}{2r^3}t'$ $+\frac{Mx}{r^3}t'^3$	$+\frac{Mxx'}{r^3}$ $-\frac{2Mxx'}{r^3}t'^2$ $+\frac{Mx'^2}{r^2}r'$	$+\frac{M}{2r^3}(xy'+yx')$ $-\frac{M}{r^3}(xy'+yx')t'^2$ $+\frac{Mx'y'}{r^2}r'$	$+\frac{M}{2r^3}(xz'+zx')$ $-\frac{M}{r^3}(xz'+zx')t'^2$ $+\frac{M}{r^2}x'z'r'$
$-\frac{My}{2r^3}t'$ $+\frac{My}{r^3}t'^3$	$+\frac{M}{2r^3}(xy'+yx')$ $-\frac{M}{r^3}(xy'+yx')t'^2$ $+\frac{Mx'y'}{r^2}r'$	$+\frac{Myy'}{r^3}$ $-\frac{2Myy'}{r^3}t'^2$ $+\frac{My'^2}{r^2}r'$	$+\frac{M}{2r^3}(yz'+zy')$ $-\frac{M}{r^3}(yz'+zy')t'^2$ $+\frac{My'z'}{r^2}r'$
$-\frac{Mz}{2r^3}t'$ $+\frac{Mz}{r^3}t'^3$	$+\frac{M}{2r^3}(xz'+zx')$ $-\frac{M}{r^3}(xz'+zx')t'^2$ $+\frac{M}{r^2}x'z'r'$	$+\frac{M}{2r^3}(yz'+zy')$ $-\frac{M}{r^3}(yz'+zy')t'^2$ $+\frac{My'z'}{r^2}r'$	$+\frac{Mzz'}{r^3}$ $-\frac{2Mzz'}{r^3}t'^2$ $+\frac{Mz'^2r'}{r^2}$

pág. 18, 17.

$$\mathcal{A}(\alpha_{ij}) = \frac{1}{2}(\alpha_{ij} - \alpha_{ji}) = \frac{1}{2}(\alpha_{ij} - \beta_{ij})$$

$$\mathcal{A}_{ij} = \mathcal{A}(\alpha_{ij})$$

$$\mathcal{A}_{i1} v^i = \left[\begin{array}{l} -\frac{Mr't'}{r^2}\left[x'^2 + y'^2 + z'^2\right] \\ -\frac{1}{2}\frac{M}{r^3}t'\left[xx' + yy' + zz'\right] \\ +\frac{Mt'^3}{r^3}\left[xx' + yy' + zz'\right] \end{array}\right]$$

$$\mathcal{A}_{i1} v^i = \left[\begin{array}{l} -\frac{Mr't'}{r^2}\left[t'^2 - 1\right] \\ -\frac{1}{2}\frac{Mr'}{r^2}t' \\ +\frac{Mr't'^3}{r^2} \end{array}\right]$$

$$\boxed{\mathcal{A}_{i1} v^i = \frac{1}{2}\frac{Mr't'}{r^2}}$$

$$A_{i3} v^i = \Big[\frac{M y' r'}{r^2} t'^2 + \frac{1}{2}\frac{M y}{r^3} t'^2 - \frac{M y}{r^3} t'^4$$
$$+ \frac{M}{2r^3}\left(x x' y' - y x'^2 \right)$$
$$- \frac{M}{r^3} t'^2 \left(x x' y' - y x'^2 \right)$$
$$- \frac{M}{2r^3}\left(y z'^2 - z y' z' \right)$$
$$+ \frac{M}{r^3} t'^2 \left(y z'^2 - z y' z' \right) \Big]$$

$$A_{i3} v^i = \Big[\frac{M y' r'}{r^2} t'^2 + \frac{1}{2}\frac{M y}{r^3} t'^2 - \frac{M y}{r^3} t'^4$$
$$+ \frac{M}{2r^3}\left[x x' y' + y y' y' + z z' y' \right]$$
$$- \frac{M}{2r^3}\left[y x'^2 + y y'^2 + y z'^2 \right]$$
$$- \frac{M}{r^3} t'^2 \left[x x' y' + y y' y' + z z' y' \right]$$
$$+ \frac{M}{r^3} t'^2 \left[y x'^2 + y y'^2 + y z'^2 \right] \Big]$$

$$A_{i3} v^i = \Big[\cancel{\frac{My'r'}{r^2} t'^2} + \cancel{\frac{1}{2}\frac{My}{r^3} t'^2} - \cancel{\frac{My}{r^3} t'^4}$$

$$+ \cancel{\frac{My'r'}{2r^2}}$$

$$- \frac{My}{2r^3}\left[\cancel{t'^2 - 1}\right]$$

$$- \cancel{\frac{Mr' t'^2}{r^2}} - \cancel{\frac{My'r'}{r^2} t'^2}$$

$$+ \cancel{\frac{My}{r^3} t'^2 \left[t'^2 - 1\right]} \Big]$$

$$A_{i3} v^i = \left[\frac{My'r'}{2r^2}\left[\cancel{2t'^2} + 1 - \cancel{2t'^2}\right] + \frac{My}{2r^3}\left[\cancel{t'^2} - \cancel{2t'^4} - \cancel{t'^2} + 1 + \cancel{2t'^4} - 2t'^2\right] \right]$$

$$A_{i3} v^i = \frac{My'r'}{2r^2} + \frac{My}{2r^3}\left[-2t'^2 + 1\right]$$

$$A_{i3} v^i = \frac{My'r'}{2r^2} + \frac{My}{2r^3}\left[-2 - 2(x'^2 + y'^2 + z'^2) + 1\right]$$

$$A_{i3}v^i = \frac{Mv'r'}{2r^2} - \frac{My}{2r^3} - \frac{My}{r^3}(x'^2+y'^2+z'^2)$$

$$\boxed{A_{i3}v^i = -\frac{My}{2r^3} - \frac{My}{r^3}(x'^2+y'^2+z'^2) + \frac{My'r'}{2r^2}}$$

$$\boxed{A_{i3}v^i = -\frac{1}{2}a_3}$$

$$\boxed{A_{i1}v^i = -\frac{1}{2}a_1}$$

$$\boxed{\boxed{A_{ij}v^i = -\frac{1}{2}a_j}}$$

$$\boxed{\boxed{A_{ij}v^j = +\frac{1}{2}a_i}}$$ (pág. 23)

$$\boxed{\sum_{ij} v^j}$$

pág. 60.

$$\sum_{1j} v^j = \left[-\frac{Mr'}{r^2} t'^3 - \frac{Mt'}{2r^3}(xx'+yy'+zz') + \frac{Mt'^3}{r^3}(xx'+yy'+zz') \right]$$

$$\sum_{1j} v^j = -\cancel{\frac{Mr'}{r^2} t'^3} - \frac{Mr'}{2r^2} t' + \cancel{\frac{Mr' t'^3}{r^2}}$$

$$\boxed{\sum_{1j} v^j = -\frac{Mr'}{2r^2} t'}$$

$$\sum_{4j} v^j = \left[-\frac{Mz}{2r^3} t'^2 + \frac{Mz}{r^3} t'^4 + \frac{M}{2r^3}\left(xx'z' + \cancel{zx'^2}\right) - \frac{M}{r^3}\left(xx'z' + \cancel{zx'^2}\right) t'^2 + \frac{M}{r^2} z'r'\left(x'^2+y'^2+z'^2\right) + \frac{M}{2r^3}\left(yy'z' + \cancel{zy'^2}\right) - \frac{M}{r^3}\left(\cancel{yy'z'} + \cancel{zy'^2}\right) t'^2 \right.$$

$$- \frac{2Mzz'^2}{r^3} t'^2 + \frac{Mz}{r^3} z'^2 \, + \Big]$$

$$\cancel{\sum_{4j} v^j = \left[\frac{Mz}{2r^3}\left[2+2x'^2+2y'^2+2z'^2-1\right] + \frac{Mr'z'}{2r^2}\right]}$$

$$\sum_{4j} v^j = \left[\begin{aligned}
&-\frac{Mz}{2r^3}t'^2 + \frac{Mz}{r^3}t'^4 \\
&+\frac{M}{2r^3}\left(\boxed{xx'z'} + zx'^2\right) \\
&-\frac{M}{r^3}\left(xx'z' + zx'^2\right)t'^2 \\
&+\frac{M}{r^2}z'r'\left(x'^2+y'^2+z'^2\right) \\
&+\frac{M}{2r^3}\left(\boxed{yy'z'} + zy'^2\right) \\
&-\frac{M}{r^3}\left(yy'z' + zy'^2\right)t'^2 \\
&+\frac{M}{r^3}\left(\boxed{zz'^2}\right) \\
&-\frac{2M}{r^3}\left(zz'^2\right)t'^2
\end{aligned}\right]$$

$$\sum_{4j} v^j = \left[-\frac{Mz}{2r^3}t'^2 + \frac{Mz}{r^3}t'^4 + \frac{M}{2r^3}\left(rr'z' + z\{x'^2+y'^2+z'^2\}\right) - \frac{M}{r^3}\left(z'rr' + z\{x'^2+y'^2+z'^2\}\right)t'^2 + \frac{M}{r^2}z'r'\left[t'^2-1\right] \right]$$

$$\sum_{4j} v^j = \left[-\cancel{\frac{Mz}{2r^3}t'^2} + \cancel{\frac{Mz}{r^3}t'^4} + \cancel{\frac{Mr'z'}{2r^2}} + \frac{Mz}{2r^3}(t'^2-1) - \cancel{\frac{Mz'r'}{r^2}t'^2} - \frac{Mz}{r^3}t'^2(t'^2-1) + \frac{Mz'r'}{r^2}\cancel{\left[t'^2-1\right]} \right]$$

$$\sum_{4j} v^j = \left\{ \frac{Mz}{2r^3}\left[\cancel{-t'^2} + \cancel{2t'^4} + \cancel{t'^2} - 1 \; \cancel{-2t'^4} + 2t'^2 \right] + \frac{Mr'z'}{2r^2}\left[1 \cancel{-2t'^2} + \cancel{2t'^2} - 2 \right] \right\}$$

$$\sum_{4j} v^j = \left[\frac{Mz}{2r^3}\left[2t'^2 - 1\right] + \frac{Mr'z'}{2r^2}\left[-1\right] \right]$$

$$\sum_{4j} v^j = \left[\frac{Mz}{2r^3}\left[1 + 2\{x'^2 + y'^2 + z'^2\}\right] - \frac{Mr'z'}{2r^2} \right]$$

$$\boxed{\sum_{4j} v^j = \left[\frac{Mz}{2r^3} + \frac{Mz}{r^3}\{x'^2 + y'^2 + z'^2\} - \frac{Mr'z'}{2r^2} \right]}$$